Common scaling laws for city highway systems and the mammalian neocortex

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Summary

Cities and the mammalian neocortex may seem to have little in common, but each is approximately a surface with a network of conduits (roads and neurons, respectively) connecting its disparate parts. Because both cities and brains are under selection pressures to make their connections efficiently, we investigate the hypothesis that the organization of city highway networks and the mammalian neocortex may be governed by common principles. Here we measure how city highway networks vary with city size and find that, consistent with the hypothesis, highway networks scale with exponents nearly identical to those found for the analogous quantities in the neocortex. As a function of surface area, the number of conduits scales approximately as the 3/4 power, the number of "leaves" (highway exits and synapses) scales approximately as the 9/8 power, propagation velocity scales approximately as the 1/8 power, and total conduit surface area scales approximately as the 11/8 power. We also find that city population scales as the 1.46 power of surface area, potentially driven by the total surface area of highways. We discuss the extent to which explanations for neocortical scaling can be extended to cities.

A variety of scaling laws are known for the mammalian neocortex relating gray matter volume, total number of synapses (Changizi, 2001, 2003), white matter volume (Frahm et al., 1982; Hofman, 1989, 1991; Prothero 1997b; Allman, 1999; Zhang & Sejnowski 2000; Bush & Allman, 2003), number of neurons (Tower, 1954; Jerison, 1973; Passingham, 1973; Prothero 1997b), surface area (Jerison, 1982; Prothero and Sundsten, 1984; Hofman 1985, 1989, 1991; Prothero 1997a), axon caliber (Changizi, 2001; Shultz & Wang, 2001), and number of cortical areas, or compartments (Changizi & Shimojo, 2005). These neocortical scaling laws appear to be a consequence of selection pressure for a sheet-like structure (namely, gray matter) to economically maintain a high level of interconnectedness (Changizi, 2001, 2003, 2007a, 2007b). We might therefore expect to find similar scaling laws for any sheet-like structure under similar selection pressures, in which case the neocortex would be just an instance of a more general kind of structure. Here we investigate city highway systems as a potential kind of network which may be driven by similar principles as the neocortex. We chose to examine city highway networks for several reasons. First, because cities lie on the land they are approximately a surface, or a sheet. Second, they are under selection pressure to efficiently interconnect via highways and roads. Third, the organization of city highway networks tends to be driven by political and economic forces over decades, rather than being planned in advanced following known principles of highway engineering (e.g., Mannering & Kilareski, 1998)—i.e., city highways systems are a result of an evolution-like mechanism. Fourth, highway network data are readily available (and our data set consists of 60 U.S. cities varying in population from 10^4 to nearly 10^7, see legend of Figure 1). Finally, the organization of city highway systems is interesting in and of itself, and better understanding them could potentially lead to better highway systems.

The number of highways in a city is prima facie analogous to the number of white-matter-projecting, pyramidal neurons in neocortex. Because cities have a tendency to be organized radially around an urban city center, we measured the number of highways as the number of radially-directed "spoke" highways plus the number of concentric "ring" highways. Combinations of highway segments were categorized as a "ring" if they tended to allow circumferential travel around the city center, and as a "spoke" if they tended to move radially away from the city center. One can easily see this spokes-and-rings organization in most of the larger cities (e.g., the map of Houston is shown above the city columns in Table 1). The number of highways increases in larger cities as approximately the 0.759 (± 0.083) power of land area (Figure 1a), similar to the exponent of 3/4 found for the number of neurons as a function of

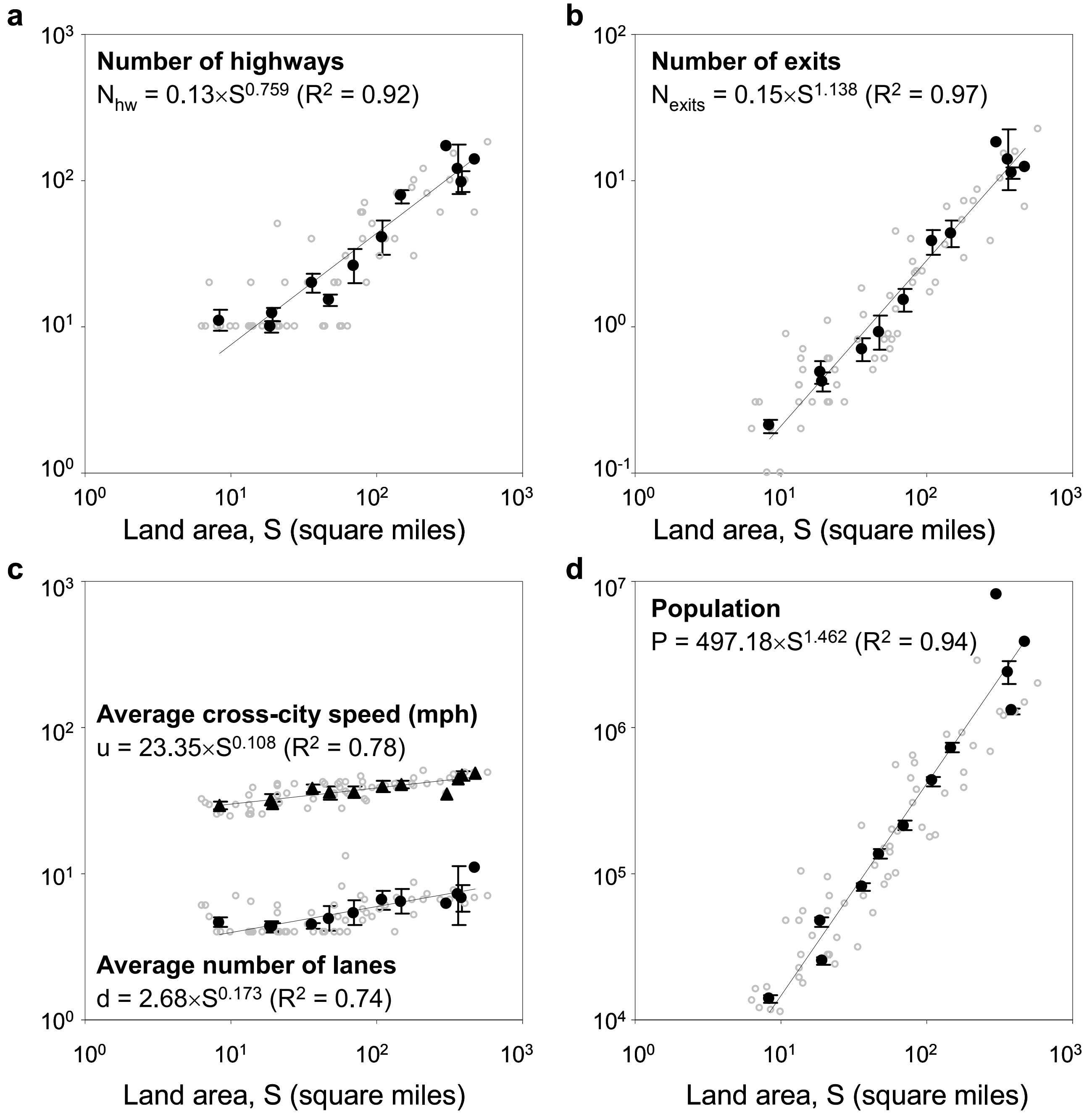


Figure 1 (a) Number of highways, (b) number of highway exits, (c) average number of lanes and cross-city (design) speed and (d) population as a function of land area. Data are from 60 U.S. cities with highways, ranging in population from 10^4 to 10^7. Best fit power-law for the binned data shown (raw data shown in light gray). The sampled cities, and bins, are as follows: population from 10^4 to $10^{4.25}$ (Los Lunas, NM; Williamsburg, VA; Amherst, OH; Berkeley Heights, NJ; Wilsonville, OR; Greenfield, IN; Canton, GA), $10^{4.25}$ to $10^{4.5}$ (Gallup, NM; Gillette, WY; Elizabethtown, KY; Helena, MT; Douglasville, GA; Cookeville, TN; Bangor, ME), $10^{4.5}$ to $10^{4.75}$ (Cape Girardeau, MO; Wausau, WI; Bellevue, NE; Tigard, OR; Battle Creek, MI; Springfield, OR; Cheyenne, WY), $10^{4.75}$ to 10^5 (Janesville, WI; Medford, OR; Santa Fe, NM; Las Cruces, NM; Albany, NY; Macon, GA; Davenport, IA), 10^5 to $10^{5.25}$ (Lowell, MA; Peoria, IL; Sioux Falls, SD; Hampton, VA; Rockford, IL; Jackson, MS), $10^{5.25}$ to $10^{5.5}$ (Little Rock, AR; Boise, ID; Spokane, WA; Durham, NC; Rochester, NY; Toledo, OH), $10^{5.5}$ to $10^{5.75}$ (Wichita, KS; Honolulu, HI; Tulsa, OK; Cleveland, OH; Albuquerque, NM; Washington, DC), $10^{5.75}$ to 10^6 (Seattle, WA; Baltimore, MD; Memphis, TN; Columbus, OH; Detroit, MI; San Jose, CA), 10^6 to $10^{6.25}$ (Dallas, TX; San Diego, CA; San Antonio, TX; Phoenix, AZ), $10^{6.25}$ to $10^{6.5}$ (Houston, TX; Chicago, IL), $10^{6.5}$ to $10^{6.75}$ (Los Angeles, CA), $10^{6.75}$ to 10^7 (New York, NY).

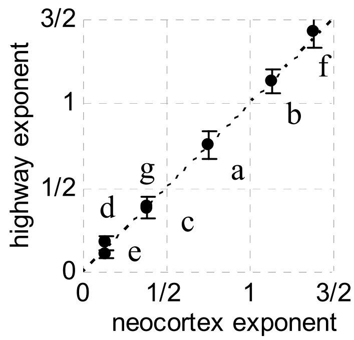
total convoluted surface area in neocortex (Table 1; exponents range from about 0.7 to 0.81; Tower, 1954; Jerison, 1973; Passingham, 1973; Prothero 1997b).

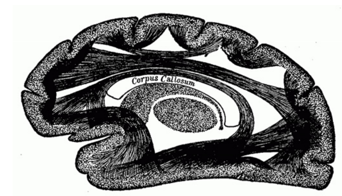
Analogous to synapses in neocortex are highway exits in city highway systems (measured as the number of exits for a unidirectional traversal of all highway stretches), and Figure 1b shows that the number of highway exits increases as about the 1.138 (95% confidence interval: ± 0.072) power of land area, or approximately as the 9/8 power. The number of exits therefore increases more quickly than land area, with surface density of exits increasing as the 0.138 power, or approximately the 1/8 power, of land area. In our data gathering we also measured the number of zip codes (using www.city-data.com) and the number of public high schools (using www.greatschools.net), and found that each scales just as does the number of exits ($N_{zip}=0.103S^{1.084\pm0.076}$, $R^2=0.97$, and $N_{pubhigh}=0.106S^{1.120\pm0.097}$, $R^2=0.95$, where S is land area), so that they, too, have similarly increasing surface density in larger cities. Some of the increasing surface density of these infrastructure-like variables is accommodated by the "thickness" of cities increasing, because larger cities tend to have taller buildings (for high schools and post offices, for example), and greater numbers of raised highways. We do not currently possess "city thickness" data (e.g., average building height) on which to test whether thickness rises slowly as approximately the 1/8 power of land area. For neocortex, it is known that the total number of synapses—which scales proportionally to gray matter volume (Abeles 1991, Changizi 2001)—scales approximately as the 9/8 = 1.125 power of total convoluted surface area (Table 1; exponents range from about 1.085 to 1.124; Jerison, 1982; Prothero and Sundsten, 1984; Hofman 1985, 1989, 1991; Prothero 1997a). This is very close to the exponent of 1.138 for the total number of highway exits, suggesting perhaps that 9/8 may be the theoretical exponent for exits (as well as number of zip codes and number of public high schools). As was the case for city highway networks, the surface density of synapses rises approximately as the 1/8 power of neocortex surface area, and this is entirely accommodated by the thickness of gray matter increasing as the 1/8 power (Jerison, 1982; Prothero and Sundsten, 1984; Hofman 1985, 1989, 1991; Prothero 1997a), amounting to a slow increase from about half a millimeter in the smallest mammals to a couple millimeters in man. Gray matter thickness may be, then, akin to the "thickness" of a city, or the degree to which cities grow in the third dimension. Other consequences of how the number of leaves and conduits scale are that for both neocortex and city highway systems (i) the number of leaves per conduit scales approximately as the 1/3 power of the number of leaves (Table 1), and (ii) the number of leaves per conduit scales approximately as the square root of the number of conduits.

The number of highway lanes (measured by sampling from each highway using maps.google.com) is prima facie analogous to the caliber of an axon, and rises as approximately the 0.174 (± 0.038) power of surface area (Figure 1c, bottom), close to an exponent of 3/16 = 0.1875. In neocortex the diameters of neuron somas and axons scale approximately as the 1/8 = 0.125 power of the total convoluted neocortex surface area (Table 1; Changizi, 2001; Shultz and Wang, 2001; Changizi, 2007a, 2007b). Highway "caliber" thus appears to scale more quickly than white matter axon caliber (Table 1). Highways are inherently two dimensional (or 1D cross section), however, whereas axons are three dimensional (or 2D cross section), and so some differences in how conduit diameter increase as a function of the number of leaves per conduit may be expected. For neocortex, the number of synapses per neuron, δ_{syn}, appears to relate to axon diameter, d_{axon}, as $d_{axon}^3 \sim \delta_{syn}$ (consistent with Murray's Law; Murray 1926; Cherniak et al. 1999; Chklovskii & Stepanyants 2003; Changizi 2003). For city highway systems, however, the number of exits per highway, δ_{ex}, appears to relate to highway diameter, $d_{highway}$, as $d_{highway}^{0.379/0.174} = d_{highway}^{2.18} \sim \delta_{syn}$. That is, rather than Murray's d^3-law holding, it appears closer to a d^2-law for highways.

We measured cross-city travel speed (measured as the road distance traveled across the city divided by the travel duration, using design speeds, averaged for two trips, one across the "long" axis of the city and the other along the

Table 1 Comparison of city highway system and neocortex exponents for quantities as a function of surface area.





Generic name	**Variable for city highways**	**City highway system**	**Variable for neocortex**	**Neocortex exponent ≈**
Surface area	Land area	1	Total convoluted surface area	1
(a) # conduits	# highways	0.759 (±0.083)	# pyramidal neurons	3/4 = 0.75
(b) Total # leaves	Total # exits	1.138 (±0.072)	Total # synapses	9/8 = 1.125
(c) # leaves per conduit	# exits per highway	0.379 (±0.064)	# synapses per neuron	3/8 = 0.375
(d) Diameter of conduit	# highway lanes	0.174 (±0.038)	Diameter of white matter axon	1/8 = 0.125
(e) Propagation velocity	Velocity of cross-city travel	0.108 (±0.021)	Propagation velocity of white matter axon	1/8 = 0.125
(f) Total surface area of conduits	Total surface of highways	1.433 (±0.096)	Total surface area of white matter axons	11/8 = 1.375
	Population	1.462 (±0.141)		
			Total volume of white matter axons	3/2 = 1.5
(g) # compartments	# concentric ring regions	0.390 (±0.055)	# cortical areas	3/8 = 0.375

"short" axis, and using a combination of Mapquest and Google Earth) as the analog of white matter propagation velocity, and found that cross-city travel speed increases as the 0.0108 (± 0.021) power of land area (Figure 1c, top). For neocortex, myelinated axon conduction velocity is directly proportional to axon diameter (Hursh 1939; Rushton 1951; Bullock & Horridge 1965), and so white matter axon conduction velocity scales as approximately the 1/8 = 0.125 power of total convoluted surface area (Table 1), which is close to the exponent for cross-city travel speed. Unlike the direct proportionality between speed and conduit diameter for neocortex conduits, cross-city travel speed scales much more slowly than highway diameter (Figure 1c, and Table 1)—namely speed, $u \sim d_{highway}^{0.108/0.174} = d_{highway}^{0.62}$ (although the slope confidence intervals are sufficiently high that neither a direct proportionality nor a square root law can be rejected).

Neocortex white matter volume scales as approximately the 3/2 power of total convoluted surface area (with exponents ranging from about 1.4 to 1.52, Table 1; Frahm et al., 1982; Hofman, 1989, 1991; Prothero 1997b; Allman, 1999; Zhang & Sejnowski 2000; Bush & Allman, 2003). The most straightforward analog of white matter volume for city highway networks would be the entire volume utilized by highways, but we do not currently possess data for highway depth, and cannot calculate volume (we do not know, for example, if highway depth scales in the same manner as width). However, highway volume may not be an interesting measure because highway traffic would appear to depend on the width of the highway (or number of lanes), not on the depth. Thus, the total surface area of highways would appear to be of interest, and the neocortical analog of this is the cumulative surface area of white matter axons. Total white matter surface area is the product of the number of neurons, the length of white matter axons, and axon diameter. Assuming axon length scales as the cube root of white matter volume, one may derive (using exponents in Table 1) that the surface area of white matter scales as the 11/8 = 1.375 power of total convoluted surface area. The total highway surface area may similarly be estimated, and assuming highway length scales as the square root of city land area, one may derive (using exponents in Table 1) that total highway surface area scales as the 1.433 (± 0.096) power of city land area, close to the 11/8 exponent for the analogous quantity in neocortex.

Population of a city scales approximately as the 1.462 power of city land area (Figure 1d, Table 1), meaning population density increases nearly as the square root of land area. How do larger cities accommodate such relatively fast increases in population density? Based on the similarity of this exponent to that for total highway surface area, one might speculate that highway surface area per population may be an invariant (although see Bettencourt et al., 2007, where population scales as the 1.205 power of road (not highway) surface area for a set of 29 German cities). That is, it suggests that rather than population being driven by city surface area, population may be being driven by the total surface area of highways, as if each person requires some fixed alotment of highway surface area (e.g., the area required by a car for safe travel). An alternative hypothesis is that perhaps population actually scales as the 3/2 power of surface area (also within the 95% confidence interval), and this could be explained by modeling the population as flowing on a surface with a central source or sink (such as the city center), where it has been shown (Dreyer 2001) that the mass of the flowing material scales as the 3/2 power of the surface area (and, more generally, as (D+1)/D where D is the dimension of the system).

An explanation for neocortical scaling concerns the economical manner in which neocortex compartmentalizes as the brain enlarges, and has been successful at predicting how the number of cortical areas, inter-area connectivity and intra-area connectivity scale with brain size (Changizi, 2001, 2003, 2005, 2007a, 2007b). Cities compartmentalize as well. For example, there tends to be a downtown business district, rather than finding these businesses uniformly distributed throughout the city. Such compartmentalization may tend to minimize costs for the infrastructure needed near businesses, as well as minimizing travel costs for business-business and business-infrastructure interactions, keeping travels short and on surface streets within each functionally specialized area. Too few compartments in a city will tend to be uneconomical because within-compartment travel will become too costly, and too many compartments will tend to be uneconomical because each compartment will tend to require a highway route to all the other areas, leading to unnecessarily high highway costs. For neocortex this tradeoff leads to the compartments increasing approximately as the 3/8 power of total convoluted surface area (Changizi, 2001, 2005). For cities it is not yet clear to us how to measure compartmentalization. Because cities tend to change their make-up as a function of radial distance from city center, one hypothesis for what city compartments might be are the concentric ring regions around the city, starting with city center. For example, the city map shown in Table 1 possesses four concentric ring regions. Interestingly, this notion of city compartments does scale approximately as the 3/8 power of land area (# ring regions ~ $0.33S^{0.390\pm0.055}$, R^2=0.88, Table 1), although whether this is an appropriate measure of the number of city compartments we do not yet know.

Cities are not brains, of course, and the metaphor can only be pushed so far. For example, whereas a single white matter axon tends to connect just two regions of neocortex, and makes no direct axon-axon connections, a single highway makes *en passant* exits all along its length, and connects directly to other highways via interchanges (# interchanges ~ $0.051S^{0.993\pm0.127}$, R^2 = 0.92, or approximately directly proportional to land area). And it is not clear what the brain analogy for people might be (possibly the material transported within axons?). Nevertheless, Table 1 and its plot on the upper left show that there are wide similarities for these two radically different kinds of network, suggesting that they are instances of a more general class of network. City highway networks may actually provide a model for better understanding fundamental properties of neocortical scaling, having the benefit of comparably infinite amounts of easily-accessible, free data.

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