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A Typology of Spatial and Temporal Scale Relations

A simple typology of relations between any two geographical scales is established by qualitatively comparing their respective grains and extents. This typology is applied to spatial, temporal, and spatiotemporal scales. It describes seven relations between any two scales in either space or time. These basic relations yield a set of 169 qualitatively different spatiotemporal scale relations, a subset of which is portrayed diagrammatically. If it is possible to transform processes or patterns from one scale in the relation to the other, up to four scaling methods may need to be simultaneously applied, depending on the relation. Scaling methods might be classified as forms of grain generalization, grain decomposition, extent extrapolation, or extent selection. This typology may also provide a framework for investigations of dependencies between scales, as well as a reference scheme for observations of scale nonstationarity. The possibility is offered that any relation that forms a nonintersecting hierarchy in either space or time is a relation between essentially independent scales. However, the use of this typology is contingent on a number of factors, and it is offered as a tool, rather than a solution, for problems of scale.

> "This being so," asked the Earl of the River, "may I take heaven and earth as the standard for what is large, and the tip of a downy hair as the standard for what is small?"
>
> "No," said the Overlord of the North Sea. "Things are limitless in their capacities, incessant in their occurrences, inconstant in their portions, uncertain in their beginning and ending. For this reason, great knowledge observes things at a relative distance; hence it does not belittle what is small or make much of what is big, knowing that their capacities are limitless."
>
> Chuang Chou, ~300 B.C.

Even when scale is the main theme of discussion, the term itself is seldom examined. The resulting absence of specificity may be intended to encourage freedom of conceptualization, but this occurs at the risk of fundamental misunderstanding. Scales are often assumed to be linearly ordered from coarse to fine, and translation across scales occurs in either a top-down or bottom-up manner (Turner, Dale, and Gardner 1989b), as reflected by the common use of terms like "scaling up" and "scaling down." The purpose of this study is to show that scales are not necessarily so

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ordered. Based on two commonly accepted components of scale (grain and extent), scale space is shown to be at least two-dimensional, and spatiotemporal scale space is four-dimensional. The derivation here is simple, but it provides a framework of comparison for alternative scaling methods and derivations of characteristic scales. Although this formalization of scale relations is vulnerable to the fragility and applicability of its underlying components, it may provide a qualitative language with which to consider questions of scale interdependencies and stationarity.

The literature of geography and related disciplines is rich in discussions of scale, offering many examples and insights. The terms "grain" and "extent" have been used quite often in descriptions of scale (Weins 1989; Turner et al. 1989a; Allen and Hoekstra 1991; King 1998). Grain is the fundamental unit by which a phenomenon is measured or described, and extent denotes the spatial area or temporal duration of phenomena. Weins (1989) has likened grain and extent to the mesh size and the overall size of a sieve. At times, grain and extent are artifacts of observation, and at other times they are attributes of phenomena. Often, the determination of their values is both important and difficult. In land-use/land-cover classification, for instance, different grain sizes yield different research results. One square meter is too fine an area to contain the complex criteria for a particular land use, and ten square kilometers may be too broad. Defining the intrinsically fundamental unit of any geographic phenomenon and connecting it to the wider extent of that phenomenon is often a tricky endeavor.

In many discussions, scale is synonymous with extent. Such a definition is usually insufficient. In order to define the scale of most phenomena, we must establish not only the outer boundaries of those phenomena in space and time, but also the less explicit inner boundaries, the size of fundamental structural units and moments that are directly relevant to their identities. Several arguments can be made for grounding scale in this manner. Linguistic ambiguities can be avoided by describing scales as fine or coarse grained, and as broad or narrow in extent, rather than as just large or small. Large *cartographic* scales imply a relatively fine spatial grain and, because of the size constraints of paper maps, rather limited spatial extents, whereas large *geographic* scales imply relatively broad extents and often depict phenomena with relatively coarse grains. The concept of scale is a modeling tool for some of us, and it is a metaphor for others. In any case, it functions as a sort of container in space or time for heterogeneous phenomena and processes, which have form and dynamics. Granularity is a common method of offering expression to this heterogeneity. Continuous functional methods of representing heterogeneity are also valid. In any case, extent alone is insufficient for providing the characterization that is often required of scale.

There are at least two forms of grain. One might be defined as the spacing between adjacent measurements (the resolution), and the other is the integration volume of each measurement. For a lattice of soil samples on the landscape, the resolution is the spacing between samples (which may vary tremendously), and the integration volume is the area of the sample itself, which is intended to represent a unit of soil, or pedon. It should be noted that in this case integration occurs horizontally on the plain. Vertical differentiation helps to characterize the soil itself and carries its own spatial scale. Integrative grain may have internal structure in terms of attributes, and it need not only represent a single integrated scalar value. If integration is carried out beyond observation by way of interpolation, we may artificially and sometimes questionably equate integration volume with resolution. Remote sensing instruments are designed to maintain an integration volume equal to the resolution, making the observed data space filling. However, the temporal resolution of remotely sensed data is far coarser than the temporal integration volume, or time constant. Only with continuous observation are the temporal resolution and the temporal integration volume equal. Deciding which version of grain is appropriate to discussions of scale would depend on whether the phenomena are space filling and/or time filling. In space filling scales of continuous phenomena, grain might refer to resolution. In scales of entities that are not space filling, grain might refer to integration volume.

Grain and extent may be intrinsic characteristics of the observed or extrinsic characteristics of the observation. For example, the intrinsic spatial grain of social phenomena might be the individual or the household. The spatial grain of census data used to observe such phenomena might be the block group, which is extrinsic to the individuals and households that census data are intended to represent. For a regional political or biophysical phenomenon, the intrinsic extent might be a state or watershed, respectively, but a political unit would form an extrinsic extent if the study is biophysical. The extent of a particular data set that is established by the arbitrary limitations of observation is also extrinsic. For many phenomena, we are presented with periodic functions that establish intrinsic temporal grains. The daily and yearly cycles of many geographic processes offer intrinsic time constants and temporal extents. Hours, minutes, and seconds may be used to quantify the extrinsic grain or extent of phenomena that are not tied to the broader temporal scales of orbital mechanics.

In the context of remote sensing image analysis, Cao and Lam (1997) have classified scales into four categories: cartographic, observational, operational, and resolution. Cartographic scale characterizes the spatial relation between a landscape and its map or physical model, and it is not directly relevant to the typology described here. Resolution scale is equivalent to grain. Observational scale corresponds to extrinsic extent. Operational scales correspond to the characteristic scales of processes, which are functions of their intrinsic extents. As Cao and Lam point out, operational scales are not necessarily equivalent to observational scales. That is, the scales over which processes operate do not necessarily correspond to the spatial extent of the observation of such processes.

An essential component of many discussions is the concept of the characteristic scale of a pattern or process. Generally, the characteristic scale is either equivalent to or derived from the *extent* of spatial or temporal values. Clark (1987), who compiled a list of characteristic scales based on extent in order to investigate interactions between climates, ecosystems, and societies, determined characteristic spatial scales by measuring the square root of areas over which phenomena occur. Clark defined characteristic time scales in terms of the temporal extent required for an *e*-fold increase in value, which is constant for expressions of exponential growth or decay. Since it is not constant for periodic functions, this *e*-fold increase is determined at a point in time halfway between the system's maximum and minimum values. However, Cao and Lam (1997, p. 66) have suggested that characteristic scales of both spatial and temporal phenomena are the scales of observation that display the greatest heterogeneity or variability. If we understand scale to consist of both grain and extent, these two definitions become complementary rather than contradictory, with Cao and Lam defining the grain and Clark defining the extent of a characteristic scale.

SCALE RELATIONS

Since we may define characteristic scales in terms of both grain and extent, what is meant by an ordering of scales, or of scaling up or scaling down? A matrix, rather than a linear ordering, is required. Presented below is a set of scale relations based on qualitative comparisons of the grain and extent of a pair of scales. Missing from this typology is any mention of scales of attributes (the extent and grain of electromagnetic wavelengths would exemplify one such scale of attributes). Inclusion of attribute dimensions beyond space and time may unnecessarily hinder understanding of the principles being described here.

In each relation, we will be comparing two spatial scales, ${}^{S}S_i$ and ${}^{S}S_j$. Each scale i is defined by its grain ${}^{S}g_i$ and extent ${}^{S}e_i$, where the superscript indicates that the scale is spatial and the subscript is used to distinguish between scales. By definition, for any single scale, we can assume that the extent must be larger than the grain:

$$\text{For all } {}^{S}S_i : {}^{S}e_i > {}^{S}g_i \,. \tag{1}$$

A simple index ν_i is derived by taking the quotient of grain and extent:

$${}^{S}\nu_i = {}^{S}e_i / {}^{S}g_i \,. \tag{2}$$

This index is equivalent to the idea of "large over small" that has been applied to measurement scales (Goodchild and Proctor 1997), and it may be equated with information content or data volume. It establishes a ceiling on the complexity or heterogeneity that may be represented within S_i.

Several qualitative relationships may exist between two scales in terms of pairwise comparisons of extent and grain. The relation between two scales $R({}^{S}S_i, {}^{S}S_j)$ must be a member of the set formed by specific pairwise comparisons, as discussed below.

H: Hierarchical

Any two scales are ordered hierarchically if and only if (iff) the grain of one scale ${}^{S}S_i$ is less than the grain of another scale ${}^{S}S_j$, *and* the extent of ${}^{S}S_i$ is less than the extent of ${}^{S}S_j$.

$$R({}^{S}S_i, {}^{S}S_j) = {}^{S}H_{i,j} \quad \text{iff } {}^{S}g_i < {}^{S}g_j \text{ and } {}^{S}e_i < {}^{S}e_j \,. \tag{3}$$

In other words, hierarchical scales are ordered by consistently increasing grain *and* extent. Relations of exclusively different grains *or* extents are not strictly hierarchical by this definition; these will be described later. In general, relations between the scales of ecological hierarchy theory (O'Neill et al. 1986; King 1998) are hierarchical. The ν_i and ν_j of the two scales may be larger, smaller, or equal to each other. In addition to the criterion for a hierarchical relation [equation (3)], we can usefully decompose this condition further by comparing the extent of one scale with the grain of another, further classifying hierarchical scales into three subtypes.

IH: Intersecting Hierarchical

If two scales form a hierarchical relation, and if the extent of the first scale is greater than the grain of the second scale, then the two scales *intersect.*

$$R({}^{S}S_i, {}^{S}S_j) = {}^{S}IH_{i,j} \quad \text{iff } {}^{S}e_i > {}^{S}g_j \tag{4}$$

The spatial scale of a mesoscale climate model whose extent is 5,000 kilometers intersects in this manner with the spatial scale of a global circulation model whose grain size is 500 kilometers.

NH: Nonintersecting Hierarchical

If the extent of the first scale is less than the grain of the second scale, then these two scales, although hierarchical, are nonintersecting and nonadjacent.

$$R({}^{S}S_i, {}^{S}S_j) = {}^{S}NH_{i,j} \quad \text{iff } {}^{S}e_i < {}^{S}g_j \,. \tag{5}$$

The scale of a microclimatic model whose extent is 1 kilometer does not intersect that of a global circulation model whose grain size is 500 kilometers, although they still form a hierarchical relation.

AH: Adjacent Hierarchical

In a hierarchical relation, the extent of the first scale might match the grain of the second scale. Such adjacent scales occur most notably when hierarchical scales are derived from one another.

$$R({}^{S}S_i, {}^{S}S_j) = {}^{S}AH_{i,j} \quad \text{iff } {}^{S}e_i = {}^{S}g_j \,. \tag{6}$$

Computational spatial data structures sometimes represent adjacent hierarchical levels at different depths in the data structure. For instance, Oliver and Webster (1986) used an unbalanced binary tree structure to measure variance in soil samples at different spatial scales. The grain at any level of the tree represented the extent of the sampling for lower levels of selected branches. Turning our attention to relations between scales that cannot be characterized as strictly hierarchical, we have four qualitatively different possibilities.

NE: Nested Extent

If the grains of two scales are approximately equal, it is possible for the extent of one scale to be nested within the extent of another.

$$R({}^{S}S_i, {}^{S}S_j) = {}^{S}NE_{i,j} \quad \text{iff } {}^{S}g_i = {}^{S}g_j \text{ and } {}^{S}e_i > {}^{S}e_j \,. \tag{7}$$

$${}^{S}NE_{i,j} : ({}^{S}\nu_i > {}^{S}\nu_j) \,. \tag{8}$$

Remote sensing context operators often use scales of nested extent. The nested scale S_j forms a window that can be used in conjunction with a convolution operator to scan the nesting scale S_i, revealing properties undetected at S_i. The scale of a nation's province has this relation with the scale of the nation if the grain of both is identical, for example, the census tract.

NG: Nested Grain

We often encounter situations where the extent is fixed but the grain size differs between scales, in a relation of nested grain.

$$R({}^{S}S_i, {}^{S}S_j) = {}^{S}NG_{i,j} \quad \text{iff } {}^{S}g_i < {}^{S}g_j \text{ and } {}^{S}e_i = {}^{S}e_j \,. \tag{9}$$

$${}^{S}NG_{i,j} : ({}^{S}\nu_i > {}^{S}\nu_j) \,. \tag{10}$$

Nested grain described the relation of spatial scales derived through the deliberate coarsening of spatially explicit data while the original extent is maintained. In such cases, variance (Moellering and Tobler 1972; Bellehumeur and Legendre 1997), entropy (Batty 1976), and diversity (Turner et al. 1989a) are usually reduced at the coarser grain, although nonlinearities can disrupt this tendency.

FN: Fully Nested

In a fully nested relation $FN_{i,j}$, the grain of ${}^{S}S_i$ is finer than the grain of ${}^{S}S_j$ while the extent of S_i is greater than that of S_j.

$$R({}^{S}S_i, {}^{S}S_j) = {}^{S}FN_{i,j} \quad \text{iff } {}^{S}g_i > {}^{S}g_j \text{ and } {}^{S}e_i < {}^{S}e_j \,. \tag{11}$$

$$ {}^{S}FN_{i,j} : ({}^{S}\nu_i > {}^{S}\nu_j) \ . \tag{12} $$

This "hyper hierarchical" relation is becoming increasingly important in our technologically dominated world, and it will be discussed later.

CO: Coincident

If the grains and extents of the two scales match, we should bear in mind that the extents and grains of the measurement scales of the state variables may not have been considered. Therefore, the two scales are coincident but not necessarily equivalent.

$$ R({}^{S}S_i, {}^{S}S_j) = {}^{S}CO_{i,j} \quad \text{iff } {}^{S}g_i = {}^{S}g_j \text{ and } {}^{S}e_i = {}^{S}e_j \ . \tag{13} $$

Exploring the Typology

Certain logical deductions can be made on the basis of this typology. For instance, if scale S_i is in a relation of nested extent with scale S_j, and S_j is in a relation of nested grain with scale S_k, then S_i is fully nested within S_k.

$$ \text{If } {}^{S}NE_{i,j} \text{ and } {}^{S}NG_{j,k} \text{ then } {}^{S}FN_{i,k} \ . \tag{14} $$

Based on the above discussion, the possible relations between any two spatial scales are summarized in Tables 1 and 2.

Equations (1) through (12) and Tables 1 and 2 can be adapted to describe a typology of temporal scale relations by replacing all occurrences of the superscript S with T. The extent of a temporal scale ${}^{T}S_i$ would thus be represented by ${}^{T}e_i$ and the grain of a temporal scale would be represented by ${}^{T}g_i$. Temporal "information content" would be denoted by ${}^{T}\nu_i$.

$$ {}^{T}\nu_i = {}^{T}e_i \,/\, {}^{T}g_i \ . \tag{15} $$

Temporal extent and grain depend on the intrinsic variability of phenomena. If the phenomena are periodic, the extent might be equivalent to the period. The grain would be fine enough to provide temporal form to the trend, but not so fine as to lose

TABLE 1
Matrix of Relations between Scales ${}^{S}S_i$ and ${}^{S}S_j$

	${}^{S}g_i < {}^{S}g_j$	${}^{S}g_i = {}^{S}g_j$	${}^{S}g_i > {}^{S}g_j$
${}^{S}e_i < {}^{S}e_j$	${}^{S}H_{i,j}$	${}^{S}NE_{j,i}$	${}^{S}FN_{i,j}$
${}^{S}e_i = {}^{S}e_j$	${}^{S}NG_{i,j}$	${}^{S}CO_{i,j}$	${}^{S}NG_{j,i}$
${}^{S}e_i > {}^{S}e_j$	${}^{S}FN_{j,i}$	${}^{S}NE_{i,j}$	${}^{S}H_{j,i}$

TABLE 2
Decomposition of the Hierarchical Relation ${}^{S}H_{i,j}$

${}^{S}e_i > {}^{S}g_j$	${}^{S}e_i = {}^{S}g_j$	${}^{S}e_i < {}^{S}g_j$
${}^{S}IH_{i,j}$	${}^{S}AH_{i,j}$	${}^{S}NH_{i,j}$

the trend in an finer variability related to phenomena occurring at a faster temporal scale (often explained away as noise). Determining temporal extents and grains may seem more problematic than determining spatial extents and grains. In fact, determining the intrinsic values of all four is difficult in many domains.

SPACE-TIME

The grain of a spatiotemporal scale might be measured in terms of the least possible rate by which a process can spread at that scale. This least rate might require the entire available temporal duration for a process to propagate from one grain of space to an adjacent grain of space, so the spatiotemporal grain ${}^{ST}g_i$ would be expressed as the quotient of the spatial grain and the temporal extent.

$$ {}^{ST}g_i = {}^{S}g_i \,/\, {}^{T}e_i \,. \tag{16} $$

The greatest possible process rate at this spatiotemporal scale would influence the entire spatial extent in a single unit of the temporal grain; this upper limit would denote the space-time extent:

$$ {}^{ST}e_i = {}^{S}e_i \,/\, {}^{T}g_i \,. \tag{17} $$

Consistent with equations (2) and (15), we might define a given spatiotemporal scale's ${}^{ST}\nu_i$:

$$ {}^{ST}\nu_i = {}^{ST}e_i \,/\, {}^{ST}g_i \,. \tag{18} $$

By substitution and rearrangement of terms, we can show that equations (16), (17), and (18) are consistent with the intuitive notion that the ν_i of a spatiotemporal scale is equivalent to the product of the spatial ν_i and temporal ν_i:

$$ {}^{ST}\nu_i = ({}^{ST}e_i \,/\, {}^{ST}g_i\,) = ({}^{S}e_i \,/\, {}^{T}g_i\,) \,/\, (\,{}^{S}g_i \,/\, {}^{T}e_i) = ({}^{S}e_i \,/\, {}^{S}g_i\,) \cdot ({}^{T}e_i \,/\, {}^{T}g_i) \,. \tag{19} $$

$$ {}^{ST}\nu_i = {}^{S}\nu_i \cdot {}^{T}\nu_i \,. \tag{20} $$

Spatiotemporal scales are characterized by spatial and temporal grain and extent. The intersection of the spatial and temporal relations between two scales ${}^{ST}S_i$ and ${}^{ST}S_j$ would define a spatiotemporal scale relation. For instance, we could define a hierarchical relation in both space and time as the intersection of the spatial and temporal scale relations:

$$ {}^{ST}R({}^{S}S_i, {}^{T}S_i, {}^{S}S_j, {}^{T}S_j) = ({}^{S}H_{i,j} \cap {}^{T}H_{i,j}) \text{ iff} $$

$$ ({}^{S}e_i < {}^{S}e_j,) \text{ and } ({}^{S}g_i < {}^{S}g_j) \text{ and } ({}^{T}e_i < {}^{T}e_j) \text{ and } ({}^{T}g_i < {}^{T}g_j) \,. \tag{21} $$

Phenomenon at characteristic scale ${}^{ST}S_j$ would be spatially broader and exist for a longer period of time than phenomena at characteristic scale ${}^{ST}S_i$. Also, phenomena as scale ${}^{ST}S_j$ would have a coarser structure and temporal grain than phenomena at scale ${}^{ST}S_i$. The spatiotemporal scale of leaf physiology and that of whole-tree physiology (King 1991, p. 481) would form the relation of $({}^{S}H_{i,j} \cap {}^{T}H_{i,j})$. But the spatiotemporal scale of whole-tree physiology and that of tree growth would form a coincident relation in space and a hierarchical relation in time $({}^{S}CO_{i,j} \cap {}^{T}H_{i,j})$.

DIAGRAMMATIC REPRESENTATIONS

The use of diagrams to represent spatiotemporal scales may have begun with oceanographer Henry Stommel (1963), who plotted the distribution of variability of current velocity, sea level, and other variables in the ocean on a space-time graph with logarithmic axes. Many subsequent examples of similar plots can be found (Clark 1987; Easterling 1997; Meyer et al. 1997). Most of these "Stommel diagrams" are intended to represent characteristic scales of phenomena in terms of their extents, but occasionally a graph representing the comparative grain size of phenomena in space-time can be found (Meetenmeyer 1989).

Let us consider instead a diagrammatic representation of scale that includes both grain and extent. A two-dimensional representation of a single spatiotemporal scale within a graph of logarithmic space versus logarithmic time might assume a form whose minimum x and y values represent spatial and temporal grain, and whose maximum x and y represent spatial and temporal extents. Relations between spatiotemporal scales would be represented by the relative placement of these forms. Figure 1 portrays the relations between two spatiotemporal scales where all relations are ordered: (i,j). Figure 2 portrays relations where the temporal relational orderings are (i,j) and the spatial relational orderings are (j,i).

The full set of relations is obtained by reversing the order of temporal scales in both Figures 1 and 2. Since ${}^{S}CO_{i,j}$ is equivalent to ${}^{S}CO_{j,i}$, the last columns of Figures 1 and 2 are identical. Since ${}^{T}CO_{i,j}$ is equivalent to ${}^{T}CO_{j,I}$, the bottom rows of Figures 1 and 2 and the bottom rows of the matrices derived by reversing the order of their temporal relations are identical. Eliminating the duplicate rows and columns, the size of the full matrix obtained from all orderings is 9x9, yielding 81 spatiotemporal relations. If the hierarchical relation H is replaced by all of its variants (IH, NH, AH), the matrix size is actually 13x13, yielding 169 qualitatively different spatiotemporal relations.

Alternatively, a spatiotemporal scale might be represented on a three-dimensional graph with the spatial grain along the x-axis, the spatial extent along the y-axis, and both the temporal grain and extent along the z-axis. A specified scale would appear as a vertical line segment whose endpoints represent the temporal grain and extent. Other methods of visualization are certainly possible, but pattern analysis need only consider the four-dimensional $({}^{S}g, {}^{S}e, {}^{T}g, {}^{T}e)$ unvisualized scale space.

(IN)DEPENDENCE

Two potential uses of this typology present themselves. On the one hand, it provides a framework for investigations of functional conversions from one scale to another or linkages between scales. On the other hand, it may offer a topological basis for comparisons of scales that are essentially "independent," by which I mean an independence of scales from one another (Phillips 1995), rather than an independence (of scale-invariant fractal patterns, for instance) from any particular scale (Atkinson and Tate 2000, p. 615).

Often, patterns and processes at different scales are assumed to be dependent on one another. By ecological hierarchy theory, wider or coarser scales approximate the boundary conditions of narrower or finer scales. These conditions constrain the behavior and dynamics of the processes occurring at the finer scales. Conversely, the finer scales contain processes that build structure at the coarser scales (O'Neill et al. 1986; King 1998). Processes at one scale create patterns at another scale (Levin 1992). Under this paradigm, scale interdependencies are ordered and linked. The order is unambiguous if the relationship is hierarchical in terms of the typology described here, but two scales may also be hierarchically ordered if the relation is nested (Allen and Hoekstra 1991, p. 60).

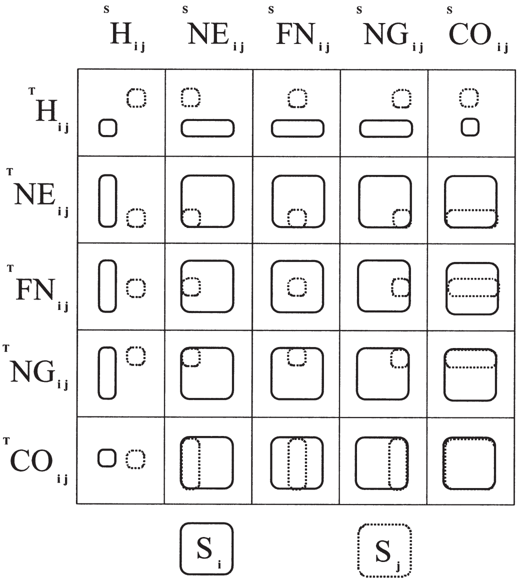

FIG 1. Spatiotemporal Scale Relations. Each matrix location represents two spatiotemporal scales on a Stommel graph modified to include grain, with logarithmic space along the *x* axis and logarithmic time along the *y* axis. Hierarchical relations are represented by the nonintersecting hierarchical (*NH*) subtype. *H* = hierarchical ($g_i < g_j$ and $e_i < e_j$); *NE* = nested extent ($g_i = g_j$ and $e_i > e_j$); *FN* = fully nested ($g_i > g_j$ and $e_i < e_j$); *NG* = nested grain ($g_i < g_j$ and $e_i = e_j$); *CO* = coincident ($g_i = g_j$ and $e_i = e_j$).

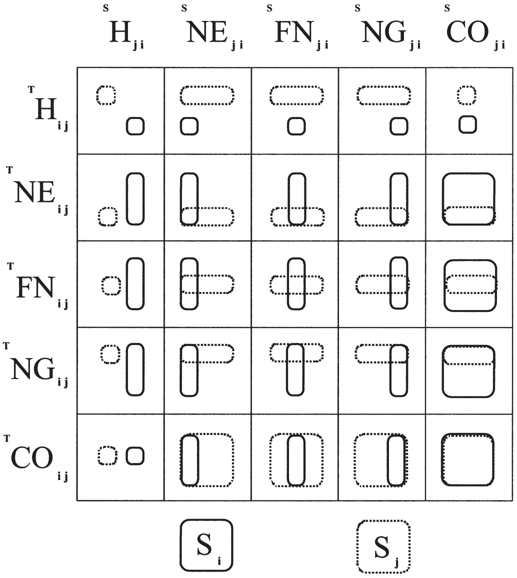

FIG 2. Spatiotemporal Scale Relations. Same as Figure 1, except that the ordering of spatial scales is reversed. *H* = hierarchical ($g_i < g_j$ and $e_i < e_j$); *NE* = nested extent ($g_i = g_j$ and $e_i > e_j$); *FN* = fully nested ($g_i > g_j$ and $e_i < e_j$); *NG* = nested grain ($g_i < g_j$ and $e_i = e_j$); *CO* = coincident ($g_i = g_j$ and $e_i = e_j$).

If different scales reflect different observational grains and extents, they may also be interdependent because they offer different parameterizations of the same underlying processes. It should then be possible to aggregate values from a fine grain to a coarse grain by taking the integral of the fine-grained values across the frequency distributions of the relevant independent variables (O'Neill 1988). However, aggregation and averaging in particular are not always appropriate methods of scaling from finer to coarser grains, so I will refer to the full set of such methods as "generalization." Other forms of scaling include "decomposition" from coarser to finer grains, "extrapolation" from narrower to broader extents, and "selection" from broader to narrower extents. Although decomposition processes are highly uncertain, they can be successful if methods exist to estimate fine-scaled values from the set of independent variables available at the coarser grain. Subpixel analysis of remote sensing data exemplifies this possibility. Extrapolation is also highly uncertain, but aggregation and selection generally are not. Scaling from S_i to S_j for each scale relation in either space or time would require one or two of these scaling methods (Table 3). Spatiotemporal scaling may require up to four scaling methods. Uncertainty is high when scaling yields an increase in ν, and uncertainty is low when ν decreases.

Scaling can be justified by either hierarchy theory (King 1991) or parameterization (Atkinson and Tate 2000). The hierarchical paradigm postulates the existence of intrinsic process linkages, whereas parameterization postulates extrinsic observational linkages. In contrast to both of these approaches is the observation that processes operating at widely separated scales often appear to be functionally independent of one another, and that phenomena at one scale may be examined and explained without recourse to phenomena at another scale (Phillips 2000). Scaling between such scales is not feasible. This condition of scale independence has been noted for instance in geomorphology (Phillips 1988, 1995) and hydrology (Beven 1995). Scales that differ from one another by two or more orders of magnitude in characteristic extent may be independent (Phillips 2000). An alternative determination is suggested by the typology discussed here. If we consider nonintersecting hierarchical scales in either space or time, it would seem likely that, given the relation $NH_{i,j}$, the extent of phenomena at S_i cannot be fully aggregated within a unit of grain at S_j, so the patterns present at S_i are independent of the processes at S_j. The thirty-two spatiotemporal relations that include a nonintersecting hierarchy in either space or time would therefore imply an independence between the two scales. However, this hypothesis is probably an oversimplification. Strongly positive feedback mechanisms and critical thresholds may allow processes occurring at one spatiotemporal scale to directly influence patterns at entirely separated, nonintersecting scales. Edge effects (Laurance 2000) and percolation thresholds (Gardner and O'Neill 1991) can also telegraph the effects of fine-grained localized processes over broad regions, directly linking nonad-

TABLE 3
Methods of Scaling from S_i to S_j

Relation	Scaling Method(s)
$H_{i,j}$	generalization and extrapolation
$H_{j,i}$	decomposition and selection
$NE_{i,j}$	selection
$NE_{j,i}$	extrapolation
$NG_{i,j}$	generalization
$NG_{j,i}$	differentiation
$FN_{i,j}$	generalization and selection
$FN_{j,i}$	decomposition and extrapolation

jacent hierarchical scales in an operational sense, under certain conditions. On the other hand, patterns and processes whose scales intersect in space and time may have no bearing on one another. The particular scale relation can only be one of the factors that determine whether patterns and processes are independent.

(NON)STATIONARITY

All of the relations in the typology have been discussed as if they were stationary. However, since patterns and processes transform through time, they may change their characteristic scales. Relations between these scales may therefore also change. Soon after publishing his diagrammatic analysis of oceanographic scales, Stommel called the assumption of statistical stationarity of processes that depend on absolute location in space and time a "desperate thing" (Clark 1987, p. 343). Nonstationarity is common to both social and physical processes. A recent theme in human geography is an apparent space-time compression that is being experienced in social realms (Harvey 1990). This compression has occurred in conjunction with an inflation of anthropogenic scales, as previous scales become fully nested within their more recent manifestations. We have seen the spatial and temporal grains of the scales of anthropogenic phenomena driven lower by technological advances while their spatial extents have grown to cover the earth. Electronic technologies have pushed grain sizes down to microns and nanoseconds, and the temporal grain of world communications is on the order of seconds. Nanotechnologies are pushing the grains of anthropogenic processes ever closer to the nanometer and the picosecond, and the strongly positive feedback process of self-replication would allow some of them to have worldwide extents (Crandall 1996).

Scale production in the social and political arenas is by nature nonstationary (Smith 1991). Let us consider a simple example in light of the typology. Individuals might have access to local government and possibly to national government, but not to international political structures that were created for governments themselves to participate in. The individual and government represent the grain and extent of one scale, and the government and international political structure represent the grain and extent of another scale. These two scales form either a nonintersecting or adjacent hierarchical relation. If, however, in a political process an entity represented at a finer grain directly influences international organizations that represent the extent over which governmental entities have traditionally communicated, then that entity will have subverted the original relation and produced two scales in a relation of nested grain, disrupting the hierarchical relation that originally existed.

In physical arenas, nonstationarity of scale may be driven by convergent and divergent influences of processes on patterns (Phillips 1999) and by bifurcations in process dynamics (O'Neill, Johnson, and King 1989). Divergent processes yield patterns that become differentiated over time, and the characteristic grains of these patterns become finer. Convergent processes have the opposite effect. The scales of processes may therefore be stationary while the scales of resulting patterns are not. Bifurcations that result when parameters cross critical thresholds may indicate the presence of dependencies between scales, as discussed above. They also imply scale nonstationarity. Hierarchy theorists conceive of scales as constraint structures, assigning them predictive power precisely because of this flexibility: "[t]he constraint structure changes through time, and the landscape may move through critical thresholds and undergo radical changes" (O'Neill, Johnson, and King 1989, p. 203). The typology here offers a number of qualitative changes that such constraint structures may experience, and it would be interesting to investigate whether such qualitative changes correspond in any way with the critical thresholds that exist for certain parameters.

CONCLUSION

As O'Neill, Johnson, and King (1989) have pointed out, geographical systems are medium-sized systems, lending themselves neither to a small number of deterministic process equations, not to the statistical stability of large-number systems. Any discussions of geographical scale and of relations between scales are therefore contingent upon differing valid conceptualizations and available data. The foundational elements of characteristic grains and extents are vulnerable to the diversity of methods that are used to define and determine them. Nevertheless, the one-dimensional ordering of scales based on a single characteristic measure and the assumption that scaling methods proceed unambiguously either up or down unnecessarily limit the expressive power of the core concepts. While this typology implies the existence of a multidimensional set of qualitatively different scale relations, it is presented here in the spirit of contingency and flexibility.

Scale remains a powerful catalyst for discovery and explanation in geography. Hindrances to understanding proceed from the limitations of language, imprecision of definition, and a selective focus on limited domains. Most geographers taking part in this discussion seem to recognize these constraints, and many are investigating topics that depend on both physical and human processes in ways that will contribute to better understanding and commonality of language in the future.

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