The impact of chaos and complexity theories in spatial analysis. Problems and perspectives

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In the past two decades, chaos and complexity theories and their related tools became extremely popular both in the hard and the soft sciences. In sociology, economy and geography, several studies were published, developing explorations in any sort of complex adaptive systems, cellular automata, random boolean networks, genetic algorithms and diffusion fractals. This paper want to assess both the potentiality and a few but significant shortcomings of this kind of studies, as they can be perceived in the current technological state of the art.

From the huge number of algorithms, theories and methods available from the complexity sciences, I'll examine only two kinds of related issues, namely fractal dimension and cellular automata modeling. In order to assess just the critical matters and not running out of space and time, I won't explain the basics of each formalism, about which the readers may learn elsewhere. So, there will be no references to Koch snowflakes, Cantor sets or the length of the coast of Great Britain this time. As I'm not a geographer, I won't deal with specific spatial issues such as formal conceptions of density, urban density gradients, temporal sets and models, changes in urban settings, and the like. This is just a generic, critical view from the computational and theoretical perspective.

Fractal dimension

Fractal analysis of cities started with *Fractal cities* by Batty and Longley (1994). In this now classical book, out of print now, they note that 'In defining the physical form of the city, its edge or boundary is the most obvious delimiter of its size and shape'. In their book, and the papers on which it is based, they calculate the fractal dimension of various towns and cities (notably Cardiff) and compare their measurements with those recorded by other researchers. In the case of Cardiff, they examine the change in its fractal dimension over time and use this as an indicator of the changing structure of the city from the late 19th century to the middle of the 20th century, based on available map data. The principal method that they use to calculate fractal dimension is known as the '*dividers method*' . Although performed in this instance using a computer program, the method is based on a manual technique originally employed by Richardson.

Using this method, Batty and Longley show how the fractal dimension of the urban boundary of Cardiff had decreased from 1.141 in 1886, to 1.117 in 1901, and 1.109 in 1992. This indicates that the boundary of Cardiff became less irregular over the time, contrary to 'the traditional image of urban growth becoming more irregular as tentacles of development occur around transport lines' (Batty and Longley 1994:185). They link the period of greatest change in fractal dimension (1886 to 1901) to the greatest changes in transportation technology.

The also analyzed the changes in the uses of land over time. One of the problems that Batty and Longley note over their analysis of the fractal dimension of different land use types is that the land parcels that they examine share common boundaries: for example, a residential district may be adjacent to an commercial/industrial zone. In defining the boundary between these two land use units they must, by definition, share a common boundary. In that sense, the measures of fractal dimension that are obtained for each land use are not strictly independent. Despite this, Batty and Longley suggest that the method remains valid and that a comparison of the fractal dimension of different land use types is instructive, both in terms of a comparison between different towns or cities and as a surrogate measure for other phenomena of interest.

After that study, a long list of fractal dimension analysis were made, applied both to contemporary data and historical maps. Today we have a lot of information about typical and not so typical fractal dimension values. The fractal dimensions of US and international cities, for instance, have values ranging from 1.2778 (Omaha]) to 1.93 (Beijing). Scientist also found that the fractal dimension of large contemporary cities tends to cluster around the latter value. Studies of urban growth of London between 1820 and 1962 show that fractal dimensions for this period vary from 1.322 to 1.791. The fractal dimensions for the growth of Berlin in 1875, 1920 and 1945 are 1.43, 1.54 and 1.69, respectively. In sum, there is no agreement between the Cardiff and the global data, but the analysts are confident that a general trend of fractal dimension change will be discovered some day.

Since the days of the first Batty studies, only ten years ago, many things happened. For the average user, the researcher and the student there are several available software options for measuring the fractal dimension: HarFA, Fractal Analysis System, Kindratenko's Fractal Analysis of Contour, Pierre Frankhauser's Fractalyse, Bar-Ilan's Fractal Dimension application, Paul Bourke's Fractal Dimension Calculator, TruSoft Benoit™, etc. There are also fractal dimension facilities embedded into some general purpose GIS software, such as Geostat Office, Exeter GS+, SpaDiS™ and others. Before putting some of them to work, I want to review some problems with fractal dimension measures.

First of all, let me point out that there is not a single way of measuring the fractal dimension of an object. There are several widely differing definitions and measuring options. Some of them are almost impossible to calculate in computers, others are rather straightforward; the best known measures are Haussdorf dimension, box counting, perimeter area, rule dimension, information dimension, mass dimension, fragmentation dimension, etc. All the dimensional methods have a more or less sound inner logic, but their results are not necessarily proportional. The are many papers and books related to this issue, but the use of the different measures is neither always clear in the non-mathematic, applied literature, nor in the implemented software.

There are several problems around the concept of fractal dimension either. It is possible to define the dimension of a set in many ways, some satisfactory and others less so. It is important to realize that different definitions may give different values of dimension for the same set, and may also have very different properties. As Falconer has pointed out, inconsistent usage has sometimes led to considerable confusion. Some authors even interpret the concept of fractal dimension inconsistently in a single piece of work (Falconer 1999: x). More seriously, the same could be said of many of the current pieces of software.

When doing a measure, the user has several dimension definitions, many precision options, many different approaches. Usually, there is not such thing as the “fractal dimension” of an object. There are no rules of thumb for chosing a quantity instead of others. Moreover, apparently similar definitions of dimension can have widely differing properties. It should not be assumed that different definitions give the same value for dimension, even for simple sets. It is necessary, though, to derive the meaning of the measure from its definition (Falconer 1999: 37).

As an example of the problems surrounding the fractal dimension concept, I’ll examine just a few cases involving the simplest of all the varieties, namely box counting. Box counting or box dimension is one of the most widely used dimensions. Its popularity is largely due to its relative ease of mathematical calculation and empirical estimation. The definition goes back at least to the 1930s and it has been variously termed Kolmogorov entropy, entropy dimension, capacity dimension, metric dimension, logarithmic density and information dimension. Box dimension is simple and its calculation is unproblematic, but its underlying maths has several undesirable consequences, such as overly different results for closely related, almost identical figures. There are some known procedures to circumvent the problems, but this make the calculation hard (as it happens with Hausdorff-Besicovich dimension).

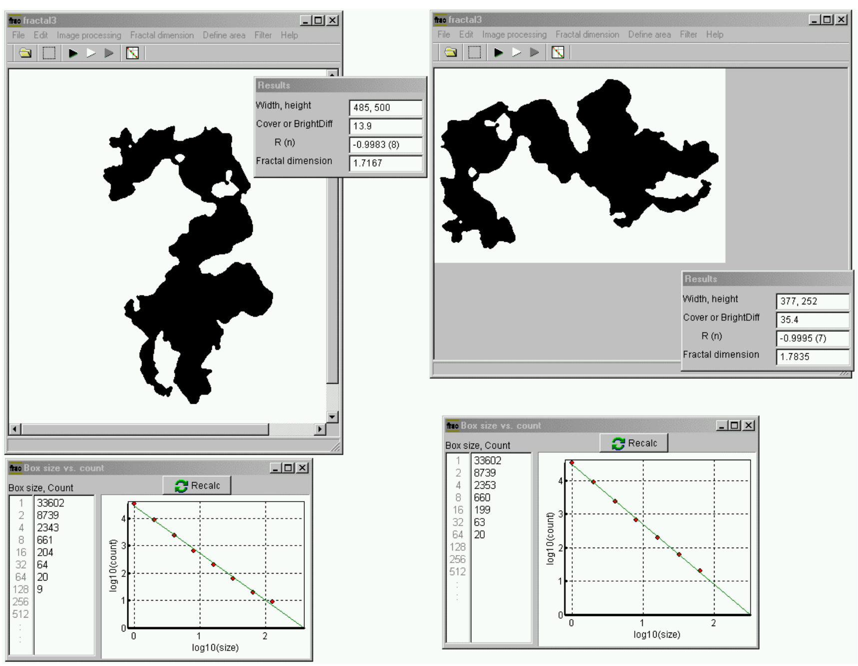

Fig. 1 – Fractal Dimension Calculator

Let’s show some consequences of this state of affairs. Fig. 1 shows Sasaki’s fractal dimension calculator applied twice to the same image; the only difference is that the second image is rotated 90 degrees. The fractal dimension for the first image was computed as 1.7167, and for the second 1.7835. It’s a rather big difference by the way, considering that fractal dimension for curves range only from more than 1 to less than 2. The implemented four digits of decimal precision is not needed, because the user gets a significant miscalculation immediately after the first one. Most of the available software, including some quite expensive pieces, deliver this sort of spurious precision stuff. For complex mathematical or computational reasons, no doubt of it, other tools behave the same wandering way when rotating, skewing, cropping blank borders or resizing an image a little bit. It's no miracle than the Cardiff and the global data do not fit.

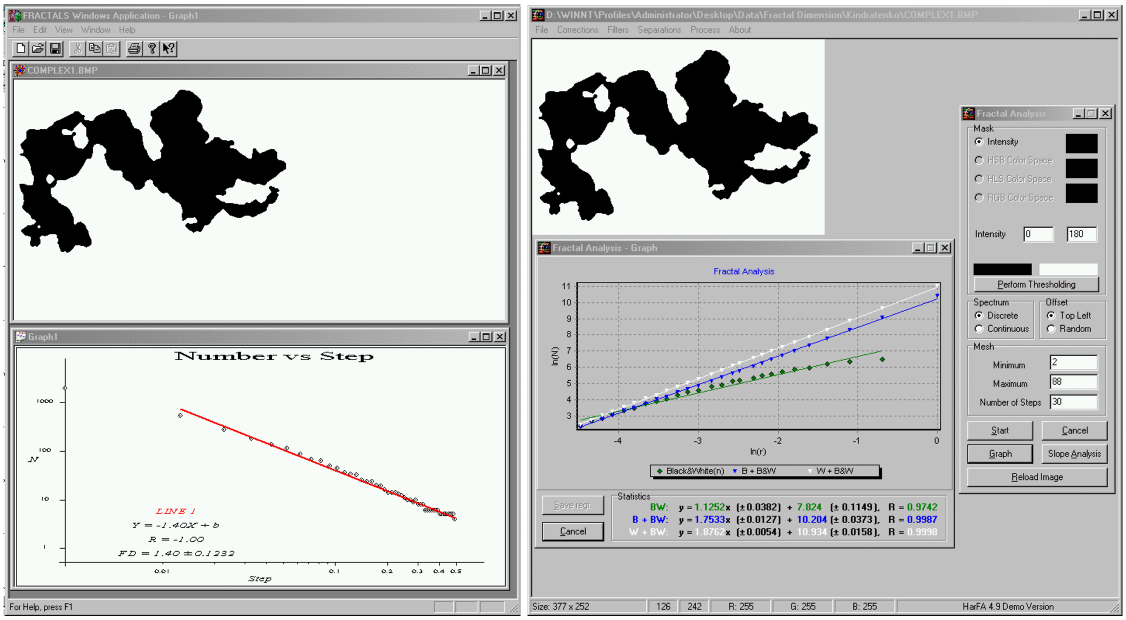

Fig.2 – Kindratenko tool and HarFA fractal dimension software

There are more to this. Fig. 2 shows at the left Kindratenko's fractal dimension analysis tool for the second sample image. In this case, the fractal dimension computed was only 1.40 ± 1.232; this involves a huge difference with the value of 1.7835 computed by Sasaki's application. Moreover, calculating the FD with HarFA (right) gives 1.7533 for B+BW and 1.8762 for W+BW. For most available applications, besides, the coloring discrimination method is not even documented. Other tests delivered even worst results: the fractal dimension of a perfect circle was evaluated as 0.9276 (it's not a fractal indeed,); for a regular square, the dimension was 1.0361. Sure, the program is confused by a closed line, but both the circle and the square should have exactly dimension 1, regardless the method used in the calculation. If a regular square is now almost a fractal, what are euclidean and non-fractal geometries all about?

In sum, fractal analysis in general (and fractal dimension in particular) are useful instruments to study the spatial organization of urban and other geographical patterns. However, the results obtained must be regarded in a comparative perspective: the single value of the fractal dimension does not supply sufficient information in order to describe the urban development of a city or place. My advice for researchers willing to apply fractal dimension in their analysis, is to spot the measure in a richer framework, carefully specifying methods and techniques, and to be aware of ill-developed and poorly documented tools.

Cellular automata modeling

Cellular automata, introduced by John von Neumann in the 1950s, have been widely used in simulation of all kinds of spatial issues: land use dynamics, regional scale urbanization and polycentricity, urban socio-spatial segregation, suburban development, location analysis, urbanism and urban growth and sprawl, complex environmental problems, percolation, segregation, polycentricity, historical urbanization, fire propagation, soil bioremediation, traffic simulation, etc.

As it is known, CA may have only four qualitatively different behaviors: fixed point, periodic, chaotic and complex (Wolfram). The four kinds of behavior (not to be dealt with here) are illustrated in Fig. 3.

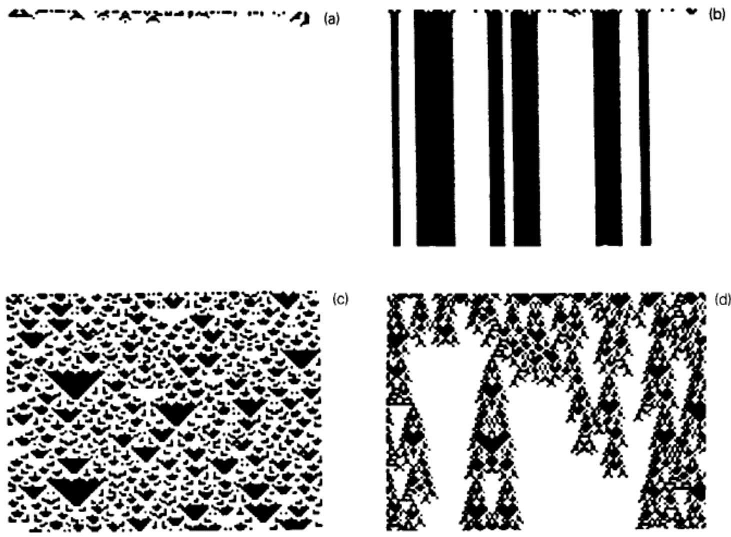

Fig. 3 – CA types, based on Wolfram

CA have many advantages for modeling urban phenomena, including their down-to-top approach, the first-order link they have to complexity theory, their connection of form with function and pattern with process, the relative ease with which model behavior and results can be visualized, their flexibility, their dynamic approach, and also their natural affinities with GIS and intra-site spatial analysis. Using CA, the geographer is fully immersed into the chaos and complexity sciences realm.

CA urban simulation models are abstract, simplified versions of real world objects and phenomena that may be used as laboratories for exploring ideas about how cities work and change over time. However, the basic CA formalism is not well suited to urban and spatial applications; the framework is too simplified and constrained to represent real cities and places. Indeed, radical modification is necessary before CA can approximate even a crude representation of an geographical system. Additional components and functionality are needed.

The good news is that in GIS and urban studies, experimentation has been prolific and innovative. The dimensions, form and structure of CA lattices have been modified and the range of cell states has been expanded. Neighborhoods have been varied and enriched considerably to accommodate action-at-a-distance and other realistic connection models beyond the simple Moore, von Neumann or Margolus types. Transition rules have also been modified far beyond the Conway Game of Life 23/3 rule, and expanded to include notions such as hierarchy, autonomous decisions, probabilistic expressions, utility maximization, loss minimization, accessibility measures, exogenous links, inertia, stochasticity, weight, land quality and cost.

By mimicking how macroscale urban structures may emerge from the myriad interactions of simple elements, CA offer a framework for the exploration of complex adaptive systems. But the basic CA simplicty has been both an strong advantage and a severe limitation. If the out-of-the-box product is too simple to represent your model and process, and if you want to add expressive power and hierarchical capabilities, the price is high. As Helen Couclelis has written (1985: 588) “all the simplifying assumptions of the basic cell-space model could be relaxed in principle: in practice of course, the result would be forbiddingly complex.” She thinks that one of the attractions of CA is the potential they provide for insights into the relationships between processes at local scales and structures at global scales. Such insight, apart from its formal and pedagogic value, also raises the possibility of a better understanding of the fundamental dynamics of spatial systems. But any insights which might be obtained are rapidly obliterated by the ever more complicated refinement of speficis model elements. CAs could be realistic and subtle by programming customized facilities and extensions; but this enrichment has a huge cost in time, money and coding complexity. Furthermore, custom tools are not commonly useful in other projects using their own semantics and considering other variables.

Other drawback of geographic CA models is that they have done relatively little to develop theory. As Torrens and Sullivan remark, claims are made that models explore various hypothetical ideas about the city, but the reported results are often more concerned with the details of model construction, at the expense of the theories that they set out to explore. Some times the power of CA modeling becomes more important than the reality being modeled. Research in urban CA modeling is becoming just that: research in modeling, and not research on urban dynamics and theory (Torrens & O’Sullivan 2001). Besides, the initial experiments with a technique or tool always seem to deliver “excellent results”, as an outcome of the well know Hawthorne effect. The excellency of the case, however, is limited to the fact that the model seems to work, and as the techniques are still on the hype the editors will accept the paper.

Another problem pointed out by the specialists is that somewhere in the transition from an abstract mathematical formalism to an spatial simulation tool, CA have evolved into a class of model that bear only transient resemblance to their parents in physics, mathematics, and computer science. The matter of modification has long been a source of disquiet in CA modeling. Again, the concern has been that model developers may focus their attention on the intricacies of model building with less attention paid to the reasons why models were developed in the first place (Idem).

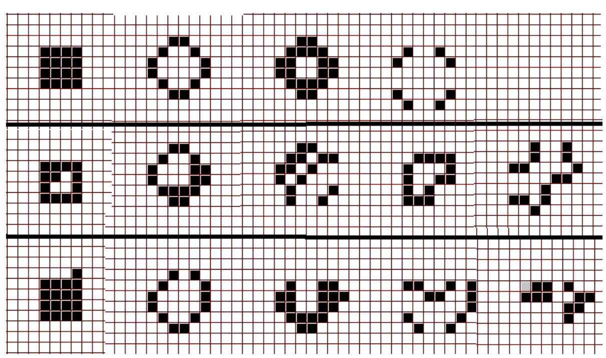

Fig. 4 – CA evolution, left to right, same rules

A most important problem, often overlook by the geographers, is that as a down-to-top complex system, CAs are bounded to something bigger than a moderate combinatory explosion. Consider just a 5x5, two states simple Game of Life board. The possible combinations are in the order of 2^{25}, or more than 33 million (actually 33,554,432). With a not too large 100x100 board, the number of possible combination grows to a number far greater than the 10^{17} seconds elapsed from the Big Bang. This means that when you try to run a model whose behavior should resemble a given process, the resemblance you are looking for may *never* occur, even if you try new combinations of rules and initial states for the rest of your life. And if the structure of states and rules falls into the chaotic or the complex types, a tiny difference will have immense consequences: this is the realm of chaos, featuring exponential sensitivy to the initial conditions, and affected by the butterfly effect.

In this regard, Fig. 4 shows how a similar pattern is transformed by the same rule after only four steps. The terminal patterns are completely different. Even if CAs are deterministic, you cannot assess the previous state of a given configuration, even knowing the transition rule. There is no formal, established, algorithmic way to get a given pattern after a number of steps. Finding a resembling pattern is, by definition, deceitful. There are no "similar" patterns or rules in nonlinear complex systems either: similarity is indeed a linear concept. In this context, it's almost senseless to try the tool as if it were a common simulation tool, running different combinations of patterns and rules, and seeing what happen. The problem space is too big, and the life is too short.

In sum, as applied to geographic systems or any other scientific area, CA are not a one-tool solution for spatial simulation; fractal growth and fractal dimension are also powerful assets for the scientist's toolkit, provided they are used with extreme care and circumspection, and that they are accurately implemented in the developed products. In most of the cases they aren't. There are no silver bullets. Specially when dealing with these advanced tools, it is advisable to extreme the precautions and think twice.

Software references

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Fractal Analysis System – Version 3.4 © 1998-2002, NILGS, NARO, coded by Hiroyuki Sasaro

Fractal Dimension – Version 1.1 © 2000, Bar-Ilan University

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