Chaos and Complexity Tools for Archaeology: State of the Art and Perspectives

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Abstract. This paper examines three powerful concepts from nonlinear dynamics, deterministic chaos, complex adaptive systems and complexity theory; they are the logistic equation, cellular automata and agent-based modeling. Some archaeological models based on these ideas will be evaluated and critically discussed.
Key words : chaos theory, complexity theory, nonlinear dynamics, cellular automata, archaeological modeling.

1 Nonlinear Dynamics: the Logistic Equation

The logistic equation is a very simple mathematical expression, but it is capable of yielding surprisingly complicated dynamics, as established by Robert May (1976), one of the pioneers of the now called deterministic chaos science. May also discovered that in the boundaries of definite ranges, a population dynamics fluctuates chaotically: the differences between equilibrium maintenance, periodicity and chaos are in the order of a few decimal values. The logistic equation is quite old; it was known as nonlinear and capricious, but nobody knew that it was paradigmatically chaotic. Pierre François Verhulst studied the equation before 1849, John von Neumann used it to generate random numbers as early as 1945, and in the 1950s Stanislav Ulam explored its weird properties, but falling short of describing it as the kind of thing later identified with chaos.

The logistic equation describes a population dynamics, as well as other phenomena responding to the same kind of regulation. The logistic equation describes not only population dynamics, but any other one variable system with chaotic potentiality. It looks like simple, but it's complex enough. The control parameter involves negative feedback; the use of the current value as a base for the next calculation involves recursion.

We are going to explain the logistic equation in a few pages, taking as departure points the logistic map description by Edward Lorenz (1995: 198-99) and an excellent example from the book of Douglas Kiel and Euel Elliott (1997); the reader will be able to test equilibrium, periodic and chaotic regimes just by using a standard spreadsheet like Microsoft Excel. The logistic equation has this form:

$$x_{t+1} = kx_t(1-x_t)$$

We are going to examine the value of a variable, x. The parameter or limit value of the formula is a constant, k. The subscript t represents time; it's the current value of variable x. Subscript t+1 represents a period of time of the variable x following the anterior, x_t. The factor (1-x_t) implements the logistic factor of limited resources. To map the formula an initial value is required; this is what in chaos theory is known as initial condition, and is represented as the first value of x_t, or x_0. Values for x_t run from 0 to 1; 0 denotes extinguished population, 1 overpopulation. Having said that, if you want to examine the dynamic behavior of the logistic equation on an spreadsheet, the initial value should be $0<x_0<1$, and the k constant should fall between 0 and 4. This constant represent the reproduction rate: if it is 0, there is no reproduction at all; is it is 4, it means that the population is reproducing at the maximum possible rate.

In the spreadsheet you can write now:

In cell A1, a fractional value for x_0 between 0 and 1. This is the initial condition.

1. In cell B1, the value of constant k, greater than 0 and less than 4.
2. In cell A2, the formula =(B1*A1)*(1-A1). This is the value of x_{t+1}.
3. Copy cell A2 until A30, for instance.
4. Generate the corresponding line graph for cells A1 to A30.
5. To modify the temporal series dynamics, just modify the values for A1 (x) and B1 (k) such as the first is any value between 0 and 1 and the second any value between 0 and 4.

A fascinating aspect of the logistic equation is that each behavior regime occurs into clearly defined mathematical limits. For instance, values of k between 0 and 3 always converge to an equilibrium situation after an initial shake. Periodic behavior starts when $k > 3$; this regime start with an oscillation which can be interpreted as a bifurcation; when the value of k is 3.5 a four-cycle period appears in a likely "way to chaos" (Feigenbaum 1978). With k equal to 3.567 an 8 period cycle appears, and incrementing k the system goes to period 16, 32, 64… until reaching chaos in the deep sense.

Chaotic behavior emerges when the values of k fall between 3.8 and 4, a tiny range indeed. What is peculiar of the chaotic regime is the lack of a repetitive pattern, or a pattern characteristically aperiodic. The famous Li and Yorke's period 3 appears clearly between 3.8284 ($1 + \sqrt{8}$) and 3.8415. In the bifurcation graphs it comes into sight as a white (or black) straight stripe in the middle of a zone of apparently random points.

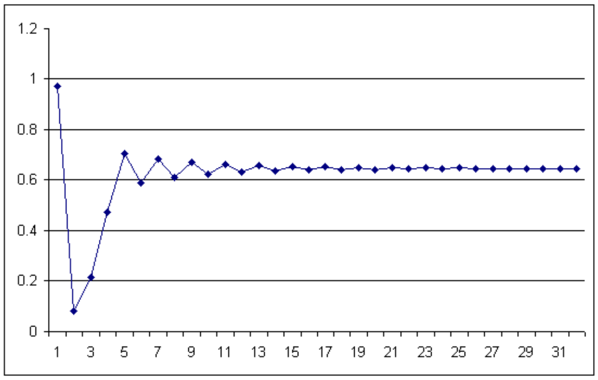

Fig. 1. Fixed point attractor

The first graph of the series shows a steady state condition with x_0 = 0.97 and k=2.827. If the initial condition were no matter how much smaller or greater, the first oscillations will be different, but in the long range the behavior stabilizes the same way. The same scenario occurs in fixed-point cellular automata and random boolean networks; in these ranges, there is an *attraction basin* with area and volume leaning toward zero.

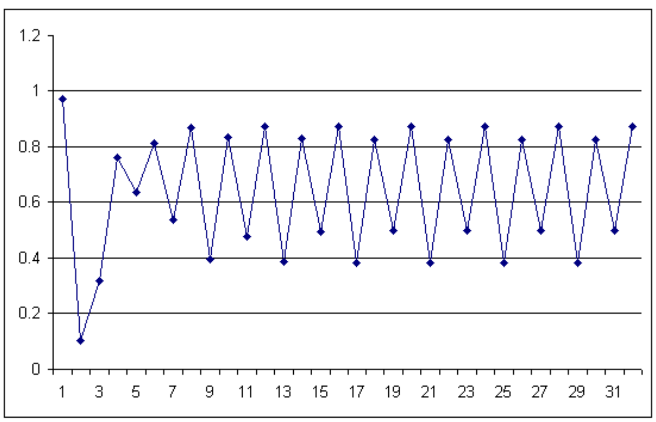

Fig. 2. Periodic attactor (period 4).

This second example illustrates a period four cyclic behavior, as when four branches open in the Feigenbaum fractal bifurcation, for values of x_0 = 0.97 and k=3.50. Small differences in the initial value would result in a shifting of cycles along time, but retaining the same cyclic structure. Once stabilized, the cycle repeats itself all the time.

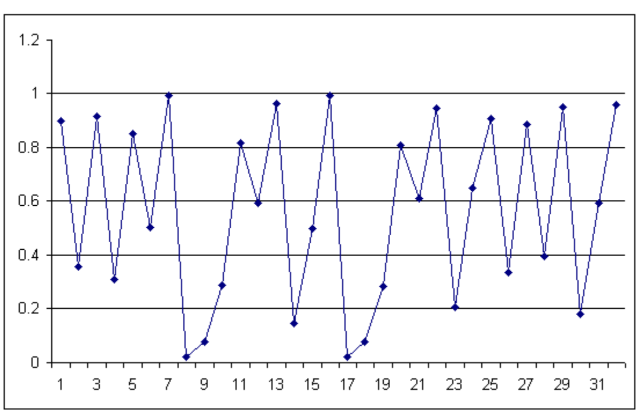

Fig. 3. Aperiodic (complex/chaotic) attractor

This is an example of aperiodic chaotic behavior for x_0 = 0.90 and k = 3.98. As in the famous Lorenz attractor, no long range sequence patterns repeats itself exactly the same way. Aperiodicity also differs from a random pattern. An aperiodic curve such as the one in the figure is a representation of the so called 1/*f* noise: human music, in whatever society (besides the Western aleatory and stochastic music) follow this kind of pattern. We want to emphasize two important outcomes of this diacronic sequence: first, that it's not possible, for a given value of *x*, to assess the following or the preceding one**;** second, that it will be still impossible to predict the next value even having knowledge of a series as long as you want. It is surprising that Gregory Bateson, ignoring almost everything about the science of chaos, depicted exactly this situation in his posthumous text (Bateson 1981: 24-25).

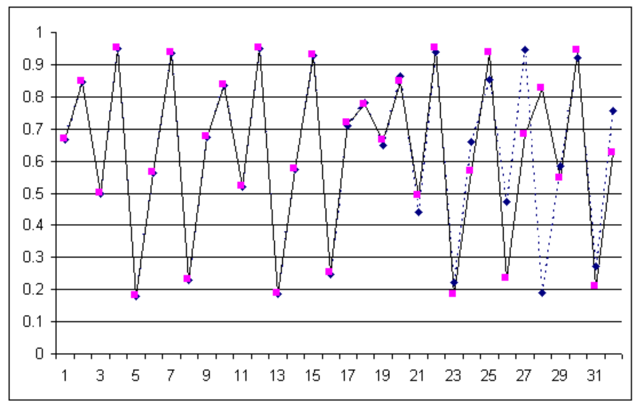

Fig. 4. Sensitivy to initial conditions

This graph depicts what in chaos theory is known as the extreme sensitivity to the initial conditions. Being k = 3.80, the solid line corresponds to an initial value of x_0 = 0.666666 and the dotted line to x_0 = 0.666333. It is demonstrated this way that when chaos conditions arise in nonlinear dynamics, it is impossible to make a long term prediction, because the values of each run differ even when the differences between two any initial values is minimal (a millionth or even less). This is the famous "butterfly effect", and an important issue for social scientists: any two systems, identical in every other respect, could develop very different stories.

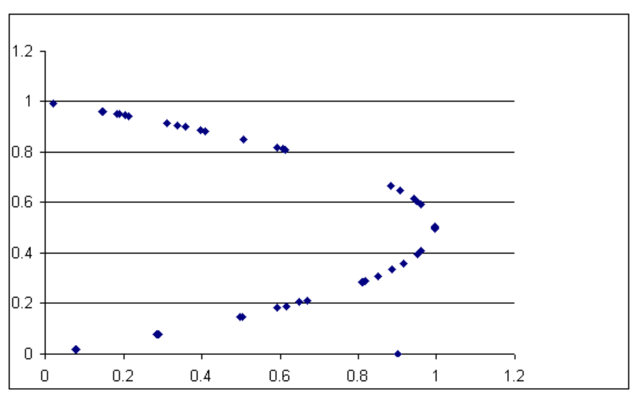

Fig. 5. Strange attractor

The last graph of the series portrays the formation of a chaotic (strange) attractor, a pattern underlying the data under examination. It's a kind of mathematical miracle: not all transitions are allowed. Even chaos has a structure, and a very specirfic one indeed. The attractor effect reveals itself plotting all the values of the series on the X axis, against the same values displaced one cell down as Y axis in an XY graph.

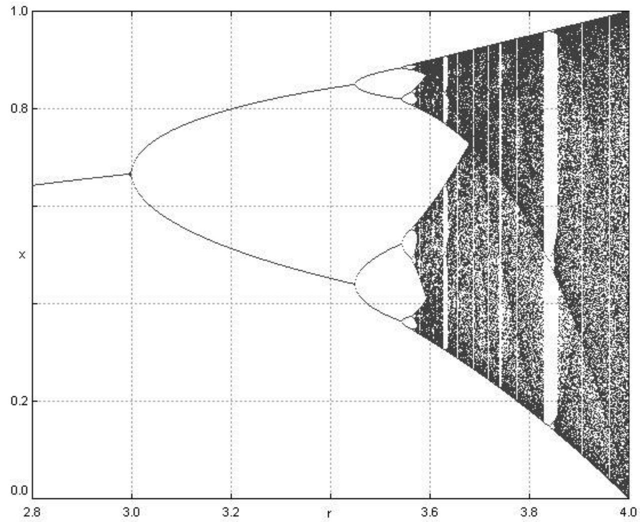

Fig. 6. Bifurcation for the logistic equation

The Fig. 6 illustrates the bifurcation graph corresponding to the logistic equation. It is a well known fractal, and as such it has several emergent properties: self-similarity, recursion, fractal dimension, period duplication determined by the universal Feigenbaum constant (4.669…), strange attractors, power-law distributions, scale independence, self-organised criticality, 1/*f* noise. There is too much stuff here to deal with in detail in a short paper like this one. Some of these properties have been studied now and then in the archaeological literature (Kohler et al 1999; Bentley and Maschner 2003); but there are not yet acceptable developments that take into account seriously the major consequences of nonlinear dynamics: namely, impredictability, emergence, extreme sensitivity to initial conditions and complex/fractal patterns. The case study literature is still too far from the state of the art of the available techniques and concepts.

2. Emergence: Cellular automata

Cellular automata (CA) are an incarnation of one of the many forms known under the name of emergent computation, defined as a pattern of behavior resulting from information processing by individual agents or cells. They are an example of how simple things produce complex behavior: a complex, adaptive system. Complex behavior emerges when a number of agents designed to behave in a certain way involve in local interactions with other agents, producing global patterns of information processing at a macroscopic level. The high-level implicit behavior emerges from the collective behavior of individuals, explicitly defined only at an individual level. Complex systems are characteristically nonlinear. Even of you know the rules of the game, and although the system is basically deterministic, there is no way of assessing what happened before the system reached a certain state. All retrodiction become impossible.

By the end of the 1960s, the british mathematician John Conway refined the description of the simplest CA capable of universal computation. The cells of the Conway's CA had only two possible states, *'on'* and *'off'* and a set of simple rules to determine the next state of the system. Conway called his system (somewhat similar to the game of Go) "the Life Game", because of the binary "dead" or "alive" state of the cells and its overall lifelike connotations.

The model of the Game of Life admitted a bidimensional representation in the form of a board. Considering as "neighbors" the eight cells that form the immediate perimeter of a cell, the rules for the time evolution of life are as follows:

(1) If a live cells has less than two neighbors, the it dies (loneliness).

(2) If a live cell has more than three neighbors, then it dies (overcrowding).

(3) If an empty cell han three live neighbors, the it comes to life (reproduction).

(4) Otherwise (exactly two live neighbors), a cell stay as is (statis).

Playing the game of life, the researcher may start from a random configuration in order to examine the classes of object than can be generated. The simplest behavior is that of the static objects no changing over time; the next class is that of the periodic, iterative objects; the third class is that of the object capable of movement, or reproduction, or both.

The third and fourth row of the Fig. 7 show the simplest mobile objects; those of the third are *glider* types, moving one slanted space in four-step processes; those of the fourth are called *fishes*. The Game of Life enthusiasts (and there are thousands of them) know a lot of these objects and patterns.

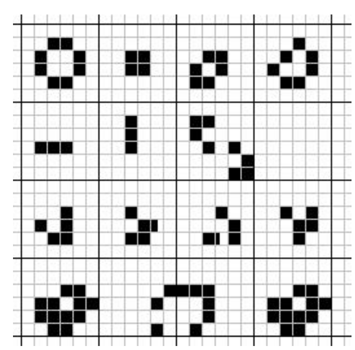

Fig. 7. Fixed, periodic and mobile objects

The dynamic behavior of periodic and ambulatory objects, the possibility of reproduction, are not issues that could be anticipated starting from the simple inspection of the rules. The visible compount objects are not fixed sets movind along a trayectory; their particles are being created and destroyed all the time. Their capabilities only exist as a product of strongly nonlinear interactions between neighboring cells, as a function of their states. Even if we restrict the attention to a pattern of 5x5 cells, no analytic procedure known so far will be able to predict the existence of, say, a gliding pattern (Holland 1998: 140). This can be discovered only by observation: a 5x5 matrix with 2 degrees of freedom has, after all, 2^{25}, that is more than 33 million of different potential configurations. Along these lines, Mathematics is now not a deductive exercise, but a experimental practice.

An interesting aspect of the CAs has to do with its tipification. The current CA taxonomies involves, by the way, a classification of levels of complexity. There are several CA taxonomies; here we are going to deal with the one proposed by Stephen Wolfram (1984). His taxonomy consists in four classes:

(1) Class I. CAs in this class always evolve to a homogeneous arrangement, with every cell being in the same state, never to change again.

(2) Class II. CAs in this class form periodic structures that endlessly cycle through a fixed numer of states.

(3) Class III. CAs belonging to this class form "aperiodic", random-like patterns that are a lot like the static white noise, with some white (or black) triangles here and there

(4) Class IV. CAs in the class form complex patterns with localized structure that mode through space in time. The patterns must eventually become homogeneous, like Class I, or periodic, like Class II; or not.

Class I automata are analogue to trivial computation programs that stop after a number of steps, or dynamic systems falling into a fixed point attractor. An attractor is simple a set of points towards which trajectories are dragged in along time. The most obvious example of a fixed point attractor is the pendulum.

Class II CAs are repetitive and reveal some likeness to infinite-loop programs, or dynamic systems characterized by oscillations within periodic or quasi-periodic boundaries. Class I and Class II automata are equivalent to regular language grammars or sofic systems, requering no memory.

Class III CAs are so extremely random that they don't display any interesting graphic pattern, but all of them have an odd trait: they are extremely sensitive to the initial conditions; if you commute a pair of cells at the beginning of a run, the subsequent behavior will be totally distinct. Wolfram has pointed the analogy of this class with context-sensitive grammars. Some of their exemplares generate random, fine grained static noise, while other produce symmetric or asymmetric fractal structures, like the Sierpińsky triangle.

Class IV CAs are by far the most fascinating. In the first place, they can execute computations, and some of them are capable of universal computation. Their diacronic evolution is also hard to describe; it is not regular, nor periodic, nor random: it has a little bit of all these types of behavior. It looks like the dynamic behavior of these CAs oscillate between chaos (random) and periodicity. They are at the *edge of chaos*, a topic too complex to deal with right now. Wolfram demonstrated that complex automata generate fractal patterns of dimension 1.59 or 1.618, and he thinks that this capability will be useful to explain the presence of self-similar structures in natural (or cultural) systems. This class includes the Game of Life, and it is analogue to Turing machines and to irrestricted languages in the Chomsky hierarchy (Wolfram 2002: 231-249).

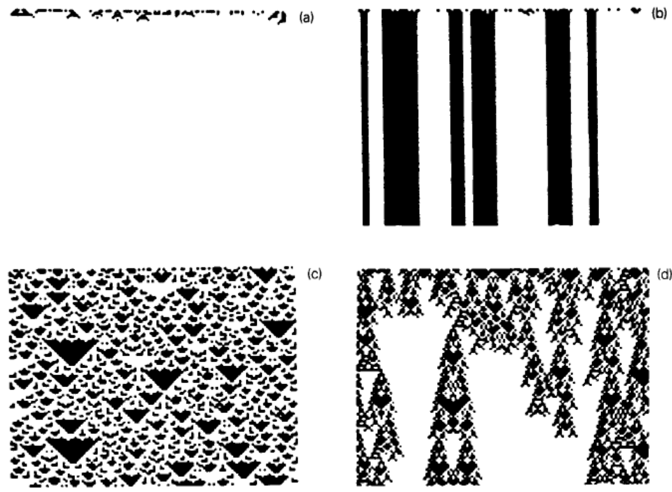

Fig. 8. The four Wolfram behaviors

The figure shows the behaviors or attractors defined by Wolfram for each one of the four classes: (a) fixed point, (b) periodic, (c) chaotic, and (d) complex, after a number of iterations starting from random initial values (Wolfram 1988). The most important issue right now is to highlight the correspondence between attractors (a), (b), and (c/d) in the CAs and the three characteristic behaviors of the distinct three ranges in the logistic equation. There are a lot more on this (all the fractal, 1/*f*, power law, self-organisation, Feigenbaum universal constant stuff); but we are running out of space.

3. Archaelogical Agent-Based Models

Agent-based models (ABM) are a natural extension of cellular automata and Stuart Kaufman's random boolean networks. In them, the space may be heterogenouos and it is not necessarily articulated in grids. The rules are also more complex, and they can change along time, conditions, events. Some probabilism could be implemented.

A classical application of ABM is Epstein and Axtell's (1996) Sugarscape. This mainly abstract implementation, with no empirical findings associated to it, is a model capable of modeling topics such as coalition formation, trade, ruled-oriented social evolution, conflict, economy and other processual phenomena. It's all a mater of semantics and interpretive imagination. Today there is a lot of work being done on artificial societies and synthetic culture based on extended CA and ABM.

One of these works is the study developed by researches of the University of Arizona and the archaeologist George Gumerman of the Santa Fe Institute, one of the notorious headquarters of chaos and complexity science. This model is designed to explain what happened in the history of the Anasazi, a tribe living in the southwest of the United States between the I and the XIV century. The main purpose was to generate an instance of an artificial culture, situating it under (virtual) environmental conditions experienced by the real Anasazi, and implement several sets of relatively simple rules in order to examine if the virtual behavior matched the actual archaeological record. Several puzzle needed to be elucidated. The Anasazi suddenly dissapeared around 1350, an embarrassing fact for archaeologists, by the way. So far, wheather and climatic changes are not persuasive as explanations. Other factors should be considered: clan formation, territorial inheritance practices, external inducements, even cannibalism. There are other problems as well: the archaeologoists have to explain why nothing happened when maize was introduced 3000 years ago, and almost nothing happened when ceramics were developed. Important social changes happened around the year 200 and nobody knows why, and there are no satisfactory explanations for the constitution of a powerful regional center between 900 and 1150, and the subsequent spectacular collapse (Dean et al 2000).

The drawing in Fig. 9 shows the contrast between the real Anasazi setlements around 1270 and its virtual reproduction in the work of Gumerman, Swedlund, Dean and Epstein (2002). The discrepancies between them, however small, prove that climatic and environmental factors are not explanatory enough. Other factors should be tested. The current "Artificial Anasazi" option in the AScape program, for instance, consider variables such as maximum and minumun age of fertility and death, basic nutritional needs, distance from the harvest areas, volume of maize produced, size of the household unit, rules of movement, metabolism and fission. No definitive solutions has been found so far, although the model is still running. No formal demarcation has been drawn between substantial and secondary factors.

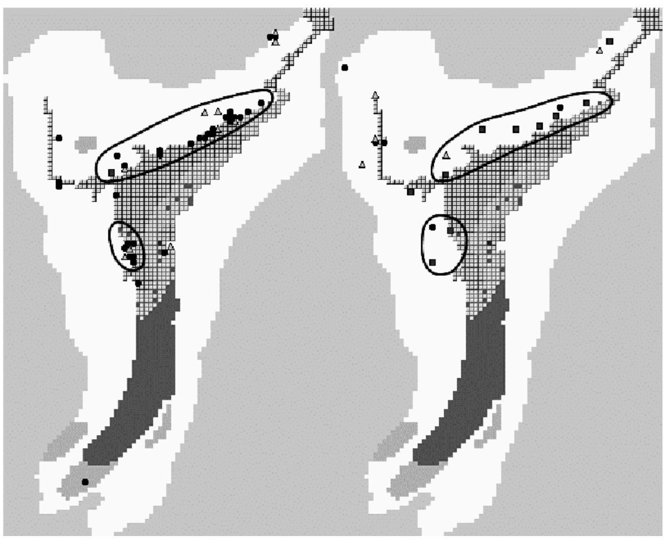

Fig. 9. Anasazi Artificial Culture

The Chaco Canyon Anasazi became the touchstone in the study of change in a remarkable transdisciplinary exploration (Lewin 1999: 1-22). The archaeologist's "inflection points" are being studied at the same level and with the same interest devoted to the biologist's punctuations, the physicist's phase transitions, the chaologist's bifurcations, the topologist's catastrophes and the cyberneticians morphogenetic processes. All phenomena somehow involve the same kind of problems. A solution found in one field could shed light on all the others.

Other scholars such as Charlotte Hemelrijk (1999) and Carlos Gershenson (2001), are studying the situation of individuals in equalitarian versus despotic societies using adaptive systems similar to random networks and cellular automata; Jim Doran and Mike Palmer (1995), are analyzing the growth of social complexity in the late paleolythic using autonomous agent models. These and other studies have little in common with the traditional system research, infused with ideas of holism and preservation of the equilibrium, such as Kent Flannery's (1986) studies on the origin of agriculture in Mesoamerica. Today's systems are not merely systems, but complex systems, built down-to-top upon the modeling of the behavior of individual agents (Bentley y Maschner 2003).

But the new models also deserve some criticism. In the first place, the search of concordance between the real life archaeological record and the behavior of the virtual model is doomed by combinatory explosion. This is implied by the 2^{25} possible outcomes of a simple 5x5, two degrees of freedom cellular automata. ¿What should be the size of a many variables, many degrees of freedom problem space? A given model could be running for centuries at lightning speed, never reaching an acceptable match.

Besides, the experimentation on cellular automata has proved that similar initial values result in very disparate global behavior, and the same is true for the logistic equation and other nonlinear models. Archaeologist, meanwhile, are constrained to work based on roughly approximate values for any variable. If any one of the underlying equations of, say, the Anasazi simulation model, falls into the range of complex aperiodic behavior, all the model will be affected by the extreme sensititivy to the initial conditions. A butterfly moving its wings in China could cause the downfall not even of the Anasazi society, but of anything else anywhere. In other words, nonlinear dynamics and cellular automata theory run against the quest implicated in the simulation models. This is what complexity and chaos science is all about. Just think about it.

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